The cylindricity of Wang tile sets

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Abstract

A Wang tile set is a finite set of unit squares where each edge got a color. A tile set T tiles the plane if the plane can be covered by $\mathbb{Z}^2$-translated copies of elements of T, where two adjacent edges must have the same color. A tile set is aperiodic if it tiles the plane, but if this can not be done in a periodic way. Most aperiodic tilings are obtained from a substitution process.

We describe a metrical invariant that quantifies the level of aperiodicity of a Wang tile set. It is relies on the ways a tile set tile can tile vertical cylinders. This invariant allows to prove that the tile sets of Kari and Culik are not ruled by a substitution process. We will also discuss the relation between the cylindricity of a tile set and the complexity of the admissible tilings.

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